

ON THE FIT OF DSGE MODELS

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Abstrakt

Cílem článku je posoudit kvalitu vyrovnání dat v modelech práce Slanicay a Vašíček (2009) pomocí jiného kritéria než posteriorní podíl šancí a srovnat výsledky s prací Slanicay a Vašíček (2009). Závěry tohoto článku jsou následující: Setrvačnost ve spotřebě je shledána důležitou a cenová indexace nedůležitou stejně jako v práci Slanicay a Vašíček (2009). Modelové varianty se zahraničním sektorem popsaným pomocí AR1 procesů dosahují vždy lepších výsledků než modelové varianty se strukturálním zahraničním sektorem. Tento závěr je v rozporu s výsledky v práci Slanicay a Vašíček (2009).

Klíčová slova

Globální analýza citlivosti, kvalita modelového vyrovnání, posteriorní podíl šancí, kvalita předpovědi, důležitost parametrů

Abstract

The goal of the paper is to assess data fit of Slanicay and Vašíček's (2009) model variants with different criteria than posterior odds and to compare the results to findings of Slanicay and Vašíček (2009). Conclusions of this article are following: Habit persistence in consumption is found important and price indexation unimportant as in Slanicay and Vašíček (2009). Model variants with foreign economy modeled with AR1 processes perform always better than structurally modeled foreign economy. This finding is in contradiction to the results of Slanicay and Vašíček (2009).

Key words

Global Sensitivity Analysis, model fit, posterior odds, forecast quality, parameter importance

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Introduction

The goal of the paper is to assess data fit of Slanicay and Vašíček (2009) model variants with different tools than posterior odds and to compare the results to findings of Slanicay and Vašíček (2009).

Sections 2.1-2.3 use tools of Global Sensitivity Analysis toolbox to deeply analyze relations within model structures that affect fit to data. Sections 3.1 and 3.2 address directly data fit by computing indices that measure data fit and quality of forecasting.¹

Final part of the paper summarizes results attained by sections preceding Conclusions.

1 Model equations

This section briefly introduces linearized model equations of all model variants that are used in the analysis. All model variants (with prior settings, data sets etc.) are taken over from Slanicay and Vašíček (2009).

For other literature with very similar models see Galí and Monacelli (2005) and Monacelli (2003). For details of linearization, see e. g. Justiniano and Preston (2004), Liu (2006), Musil and Vašíček (2006) or Remo and Vašíček (2008).

Martin Slanicay kindly provided me with Matlab codes to all model variants, which ensures comparability of results.

1.1 Denotation details

All variables are introduced as a logarithmic deviation from steady state, formally written $x_t = \log X_t - \log X$, where X is a value at steady state. Subscript t at a variable denotes relative time. Symbol E is a rational expectations operator. Symbol Δ denotes first difference so that e.g. $\Delta x_t = x_t - x_{t-1}$. AR1 shocks are denoted by ε s. Exogenous processes are denoted by ζ s. Greek letters without t subscripts denote model parameters.² Denotation of model's variables is explained in section 1.2.

Variables and parameters with a star superscript ($*$) denote foreign variables or corresponding parameters. Variables and parameters with a H subscript ($_H$) relate to home-produced goods, whereas variables and parameters with a F subscript ($_F$) relate to imported goods³.

¹ Sections 3.1 and 3.2 does not use Global Sensitivity Analysis toolbox.

² Exact meaning of all the parameters is not listed for two reasons. It is not vital for understanding paper's results and also to conserve space. Important parameters are discussed in section 1.4. Those interested in economic meaning of other (or all) parameters may consult Slanicay and Vašíček (2009).

³ Described in a more elaborate way, F subscript denotes foreign-produced home-consumed goods (in another words, imported goods).

Model consists in seven observable variables: y_t and y_t^* are modeled as (HP filter-) detrended log real GDP per worker for CR and EU12, respectively. π_t and π_t^* are modeled as demeaned quarter-on-quarter inflation rate for CR and EU12, respectively. i_t and i_t^* are modeled as demeaned nominal interest rate for CR and EU12, respectively. q_t is modeled as (HP filter-) detrended log real exchange rate. All data are from Eurostat.

Notation in pictures lacks LaTeX/MathType characters, but the paraphrasing is mostly straightforward.⁴ One notable exception is in Picture 3: First seven entries from the left denoted as E_G, E_A, E_S, E_M and ESTAR_A, ESTAR_M, and ESTAR_G are exogenous processes (ζ s) for domestic economy (E_) and exogenous processes (ζ^* s) for foreign economy (ESTAR_). Another notable exception is in Picture 2: The legend for this picture lists all seven observable variables. Y_gapcz is y , INF_gapcz is π and R_gapcz is i . Legend entries ending with “eu” are simply foreign counterparts. Last observable variable denoted RSK_gap is real exchange rate q .

1.2 Domestic block

Goods market clearing condition is $(1-\alpha)c_t = y_t - \alpha\eta(2-\alpha)s_t - \alpha\eta\psi_{F,t} - y_t^*$, where law of one price gap is defined as $\psi_{F,t} = (e_t + p_t^*) - p_{F,t}$, c is consumption, y is output, s are terms of trade, y^* is foreign output, e is nominal exchange rate, p^* is foreign price index, $p_{F,t}$ is (domestic) price index of foreign goods.

$\Delta s_t = \pi_{F,t} - \pi_{H,t}$, where π is inflation (e.g. $\pi_{H,t}$ is inflation of home-produced goods).

Domestic firms price setting equation is $\pi_{H,t} - \delta_H \pi_{H,t-1} = \beta E_t(\pi_{H,t+1} - \delta_H \pi_{H,t}) + \theta_H^{-1}(1-\theta_H)(1-\beta\theta_H)mc_t$, where mc are firm's real marginal cost that follow equation $mc_t = \varphi y_t - (1+\varphi)\varepsilon_{a,t} + \alpha s_t + \sigma(1-h)^{-1}(c_t - hc_{t-1})$

Real exchange rate definition is $q_t = e_t + p_t^* - p_t = \psi_{F,t} + (1-\alpha)s_t$.

Importers price setting equation is $\pi_{F,t} - \delta_F \pi_{F,t-1} = \beta E_t(\pi_{F,t+1} - \delta_F \pi_{F,t}) + \theta_F^{-1}(1-\theta_F)(1-\beta\theta_F)\psi_{F,t}$

Uncovered interest parity condition is $(i_t - E_t \pi_{t+1}) - (i_t^* - E_t \pi_{t+1}^*) = E_t \Delta q_{t+1} + \varepsilon_{s,t}$, (with using $\Delta e_t = \Delta q_t + \pi_t - \pi_t^*$)

Complete market assumption equation is $c_t - hc_{t-1} = y_t^* - hy_{t-1}^* + \sigma^{-1}(1-h)[\psi_{F,t} + (1-\alpha)s_t] + \varepsilon_{g,t}$,

⁴ Paraphrasing of the most important parameters is: theta_h is θ_H , theta_f is θ_F , rho is ρ_i , rho_s is ρ_s , rhostar_g is ρ_g^* and rhostar_a is ρ_a^* .

Identity for inflation definition is $\pi_t = \pi_{H,t} + \alpha \Delta s_t$. Domestic block is closed with modified Taylor rule $i_t = \rho_i i_{t-1} + (1 - \rho_i)[\psi_\pi \pi_t + \psi_y y_t] + \zeta_{M,t}$, where i is nominal interest rate, and AR1 processes $\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \zeta_{g,t}$, $\varepsilon_{a,t} = \rho_a \varepsilon_{a,t-1} + \zeta_{a,t}$, and $\varepsilon_{s,t} = \rho_s \varepsilon_{s,t-1} + \zeta_{s,t}$.

1.3 Foreign block

There are eight variants of description of foreign sector. Four variants model foreign economy with structural terms (these are called “Monacelli”), another four variants describe foreign economy with AR1 processes (variants called “VAR”).

1.3.1 Monacelli

Structural relations representing basic behavioral characteristics of a foreign economy are natural counterparts of domestic-block equations:

$$y_t^* - h y_{t-1}^* = E_t(y_{t+1}^* - h y_t^*) - \frac{1-h}{\sigma} (i_t^* - E_t \pi_{t+1}^*) + \varepsilon_{g,t}^* - \varepsilon_{g,t+1}^*,$$

$$\pi_t^* - \delta_* \pi_{t-1}^* = \beta E_t(\pi_{t+1}^* - \delta_* \pi_t^*) + \theta_*^{-1} (1 - \theta_*) (1 - \beta \theta_*) m c_t^*,$$

$$m c_t^* = \varphi y_t^* - (1 + \varphi) \varepsilon_{a,t}^* + \sigma (1 - h)^{-1} (y_t^* - h y_{t-1}^*)$$

$$i_t^* = \rho_i i_{t-1}^* + (1 - \rho_i)[\psi_\pi \pi_t^* + \psi_y y_t^*] + \zeta_{M,t}^*,$$

$$\varepsilon_{a,t}^* = \rho_a \varepsilon_{a,t-1}^* + \zeta_{a,t}^*$$

$$\varepsilon_{g,t}^* = \rho_g \varepsilon_{g,t-1}^* + \zeta_{g,t}^*.$$

1.3.2 VAR

Foreign sector is described just by AR1 processes:

$$y_t^* = \omega_y y_{t-1}^* + \zeta_{y,t}^*$$

$$\pi_t^* = \omega_\pi \pi_{t-1}^* + \zeta_{\pi,t}^*$$

$$i_t^* = \omega_i i_{t-1}^* + \zeta_{i,t}^*,$$

1.4 Model variants

As was mentioned above, the analysis uses eight model variants. They differ in the way the foreign economy is modelled and restrictions that are placed on certain parameters. Four model variants use structural “Monacelli” description (variants M1, M2, M3 and M4). Remaining four model variants describe foreign economy behavior with AR1 processes (variants called V1, V2, V3 and V4).

The original study (Slanicay and Vašíček (2009)) investigated the relevance of presence of habit persistence (parameter h) and price indexation (parameters δ) in a way of allowing the parameters to be non-zero or fix them at zero value (and eliminating them effectively from the system). Slanicay and Vašíček (2009) then compared model fit with bayesian posterior odds ratio.

Model restrictions for eight model variants are the same as in original paper Slanicay and Vašíček (2009) and are stated in Table 1. For example, variant M2 is a model with structural foreign economy and allowed habit persistence. For another example, variant V4 is a model with foreign economy modeled as AR1 processes and with allowed habit persistence and price indexation.

	M1	M2	M3	M4	V1	V2	V3	V4
restriction	$h=0$	$\delta_H=0$	$h=0$	-	$h=0$	$\delta_H=0$	$h=0$	-
	$\delta_H=0$	$\delta_F=0$			$\delta_H=0$	$\delta_F=0$		
	$\delta_F=0$	$\delta_*=0$			$\delta_F=0$			
	$\delta_*=0$							

Table 1: Restrictions imposed on parameters in various model variants

2 Global Sensitivity Analysis

This section presents results of Marco Ratto's Global Sensitivity Analysis (GSA) toolbox⁵ applied on models of Slanicay and Vašíček (2009). Following subsections 2.1, 2.2, and 2.3 present results of separate GSA tools in a summarized manner. For cross-reference and exemplary purposes, all subsections present an example of actual output of GSA toolbox prior to summarization (2.1.1, 2.2.1, and 2.3.1).

2.1 Stability analysis

Stability mapping helps to detect parameters X_i that are responsible for possible "bad behavior" of the model. Without burying into theoretic details (see Saltelli (2008), Ratto (2008) or Čapek (2009)), the use is following: „Bad behavior“ is either instability (model solution is unstable) or indeterminacy, both possibilities meaning that the solution of the model cannot be used for further needs. Stability mapping detects which parameters, and on which domain, cause the solution of the model to be „bad“. Researcher can then suitably adjust prior space so that the instability/indeterminacy regions are eliminated.

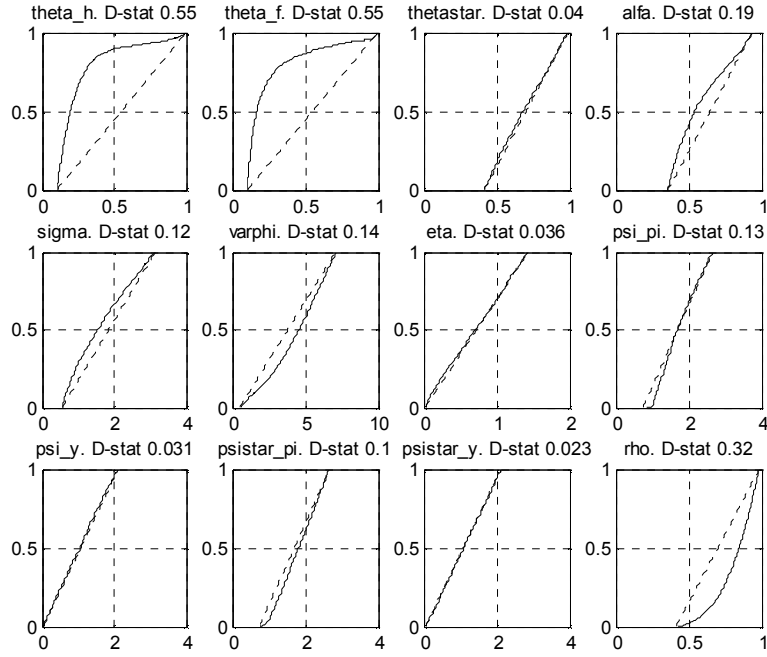
Table 2 and Table 3 introduce results of stability mapping in columns two and three. Column 2 (stability region) separates the prior space into behavioral „good“ part and non-behavioral (unstable and indeterminacy) „bad“ parts. Models with structural description of foreign economy (M1-M4) exhibit that only some $\frac{3}{4}$ of the prior space is stable. Models with VAR-foreign economy are 10 percentage points better off with approximately 86 % of prior space stable. One possible solution to this unsatisfactory situation (one that is offered by GSA) is demonstrated in the subsection 2.1.2.

⁵ The toolbox is available online at <http://eemc.jrc.ec.europa.eu/Software-DYNARE.htm>, the tools are described in Saltelli (2004 and 2008) and Ratto (2008 and 2009).

Column 3 of Table 2 and Table 3 describes, which parameters (and in which direction) influence the solution of the model. Parameters mostly responsible for unstable model are θ_H and θ_F in their lower range. Parameters creating indeterminacy include reaction parameters in (domestic and foreign) Taylor rules for inflation (ψ_π and ψ_{π^*}), if they are lower than 1.

2.1.1 Example of results from Ratto's GSA toolbox

An example of GSA toolbox results is in Picture 1, which is for model M1 and for unstable results.



Picture 1: Example: Stability analysis results for model M1, unstable region

In short, the underlying computation is following:⁶ N Monte Carlo simulations are run over prior domain, which results in two subsets, $(X_i | B)$ of size n and $(X_i | \bar{B})$ of size \bar{n} , where $n + \bar{n} = N$. The two sub-samples may come from different probability density functions (PDFs) $f_n(X_i | B)$ and $f_{\bar{n}}(X_i | \bar{B})$. Corresponding cumulative distribution functions (CDFs) are $F_n(X_i | B)$ and $F_{\bar{n}}(X_i | \bar{B})$.

If $F_n(X_i | B)$ and $F_{\bar{n}}(X_i | \bar{B})$ differ for a given parameter X_i , the parameter may drive bad behavior of the model if its value falls within \bar{B} subset. The shape of $F_{\bar{n}}(X_i | \bar{B})$ indicates, whether rather smaller or higher values of X_i drive the non-behavior. If the non-behavior CDF

⁶ Used notation is: X_i is i -th parameter, B is behavioral subset (part of domain that produces desirable results), \bar{B} is non-behavioral subset (part of domain that produces undesirable results – instability or indeterminacy).

is to the left from behavior CDF, it indicates that rather smaller values of X_i are more likely to drive non-behavior. On the other hand, if the non-behavior CDF is to the right from the behavior CDF, it suggests that rather bigger values of X_i drive non-behavior.

Model	Stability region	Stability analysis	Mapping the fit	Parameter importance
M1	76 % S 2% U 22 % I	<u>unstable</u> θ_H, θ_F lower α slightly lower ρ_i slightly higher <u>indeterminacy</u> $\psi_\pi, \psi_{\pi^*} < 1$	ρ_s higher: y, π, i, q ρ_{a^*} higher: i^* ρ_{a^*} lower: y^*, π^*	unimportant ρ_g, ρ_{g^*} most important θ_H, φ
M2	76.3 % S 1.4 % U 22.3 % I	all the same as model M1	h higher: y ρ_{a^*} higher: i^* ρ_{a^*} lower: y^*	unimportant ρ_a, ρ_s most important θ_H, φ
M3	75.1 % S 0.9 % U 24 % I	<u>unstable</u> very small part – hardly recognizable <u>indeterminacy</u> $\psi_\pi, \psi_{\pi^*} < 1$	δ_F higher i ψ_{π^*} lower all but π, i ψ_{y^*} higher all but π, y ρ_s higher: π, y	unimportant ρ_g, ρ_{g^*} most important $\theta_H, \theta_F, \theta_*$
M4	75.8 % S 0.5 % U 23.7 % I	all the same as model M3	δ_F higher i ψ_{y^*} lower q ρ_{a^*} lower π^*	unimportant ρ_g most important θ_H, h

Table 2: Global Sensitivity Analysis results, Monacelli model variants

Cumulative probability density functions shifted to the left off the dashed line in first two panels correspond to the observation in Table 2 that lower ranges of θ_H and θ_F are responsible for unstable results. Similar pictures were drawn for both instability and indeterminacy and for all eight models, the results are summarized in Table 2.

2.1.2 Expanding stability region

Most of the models demonstrate prior space, of which as little as just $\frac{3}{4}$ is stable. Global Sensitivity Analysis can help with this problem. I'll show the procedure on model M1, which exhibits 76 % of prior space stable, 2% unstable and 22 % correspond to indeterminacy. Stability analysis suggests that ψ_π or/and $\psi_{\pi^*} < 1$ cause indeterminacy. It also suggests that low ranges of θ_H and θ_F , slightly lower ranges of α and slightly higher ranges of ρ_i all contribute to unstable results.

The solution to the problem is to truncate prior densities at determinacy region. Parameters ψ_π and ψ_{π^*} have both prior value 1.5 with standard deviation 0.15. With these values, it is very unlikely that the estimation procedure could look for optimal values below 1. We can cross-check the guess by looking at the real estimate, which is approximately 1.36 and

1.38, respectively. Shifting the lower bound of the truncation (from original 0.0001 to – say – 1) elegantly reduces the part of prior space which corresponds to indeterminacy.

Model	Stability region	Stability analysis	Mapping the fit	Parameter importance
V1	85.7 % S 2.3 % U 12 % I	<u>unstable</u> θ_H, θ_F lower ρ_i slightly higher <u>indeterminacy</u> $\psi_\pi < 1$	ρ_s higher y ω_{y^*} higher y^* ω_{π^*} lower π^* ω_{i^*} higher i^*	unimportant $\rho_g, \rho_a, \omega_{y^*}$ most important θ_H, ψ_π
V2	86 % S 1.3 % U 12.7 % I	all the same as model V1	ρ_s higher y, π ω_{y^*} higher y^* ω_{π^*} lower π^* ω_{i^*} higher i^*	most important θ_H, h
V3	86.5 % S 0.8 % U 12.7 % I	<u>unstable</u> very small part – hardly recognizable <u>indeterminacy</u> $\psi_\pi < 1$	δ_F higher i ρ_g higher π, i ρ_s higher y ω_{y^*} higher y^* ω_{π^*} lower π^* ω_{i^*} higher i^*	unimportant ρ_g most important $\theta_H, \rho_i, \theta_F$
V4	86.4 % S 0.5 % U 13.1 % I	all the same as model V3	δ_F higher i ρ_s higher y ω_{y^*} higher y^* ω_{π^*} lower π^* ω_{i^*} higher i^*	most important θ_H, ρ_i, h

Table 3: Global Sensitivity Analysis results, VAR model variants

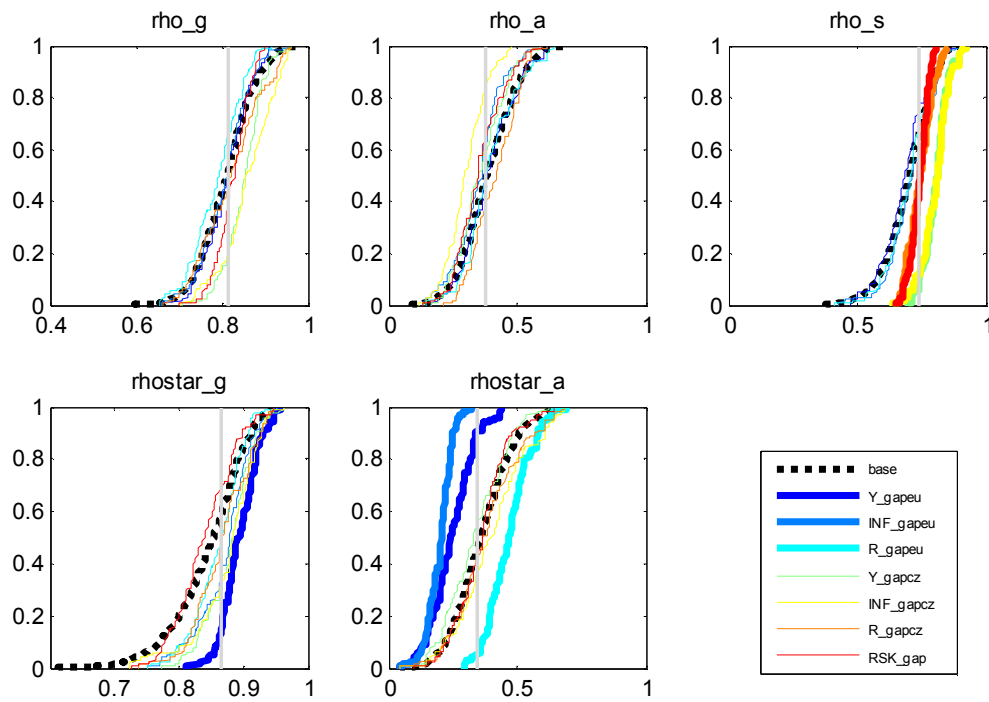
The procedure described for a case of indeterminacy is similar to the case of unstable results. As was mentioned above, low ranges of θ_H and θ_F tend to create unstable results. Both of these parameters have prior values 0.7 with standard deviation 0.1. Posterior estimates are higher than prior value (0.73 and 0.79, respectively), we can therefore shift the lower bound of truncation from original 0.0001 to 0.45. Again, the shape of prior density makes it almost impossible for the estimation algorithm to look at values as low as 0.45. Another parameter (partially) responsible for unstable results is parameter α . Prior value is 0.7 with even smaller standard deviation, 0.05.⁷ Truncation of the prior density can therefore start at 0.55. Last parameter of interest is ρ_i , but there is little we can do about its prior density. Slightly higher values result in unstable results and, indeed, posterior estimates of ρ_i are very high (0.94 on (0.0001;0.999) interval).

⁷ Slanicay and Vašíček (2009) actually state that parameter α is calibrated at value 0.7. There has probably been a minor change in versions of the models. Either way, if α was really calibrated, it wouldn't add to the prior space at all and wouldn't be subject to stability analysis.

Carrying out just these five truncations of redundant prior space results in very favorable shifts in the structure of the prior space. The final prior space consists in 95.4 % of stable results, 0.3 % of unstable results and 4.3 % of indeterminacy. This means an improvement of 19.4 percentage points in stable results. Unstable results are reduced almost 7 times and indeterminacy region is now a fifth of what it was.

2.2 Mapping the fit

Since DSGE models consist of a number of observed variables, which should fit the data as well as possible, mapping the fit may be a useful tool to learn about the linkages that drive the fit of trajectories of particular variables to data. Information provided by the results of mapping the fit can be used to unveil possible trade-offs and maybe also amend model structure or calibrate parameters properly in order to increase the fit of variables of interest.



Picture 2: Example: Mapping-the-fit results for M1 model, selected 5 parameters

Column 4 of Table 2 and Table 3 introduce results of mapping-the-fit analysis. Again, without technical details (those interested in details may consult see Saltelli (2008), Ratto (2008) or Čapek (2009)), the interpretation of the results is as follows: Let's use again model M1 for explanation. Corresponding cell (Table 2, column 4, row 2) lists 3 conflicts in data fit. “ ρ_s higher: y, π, i, q ” means that the four mentioned observable variables would prefer higher values of parameter ρ_s than its posterior distribution in order to fit data as well as possible. Because there are 7 observables, this result might seem odd, because “only” 3 observables shift posterior distribution towards lower values whereas 4 observables would prefer higher values. Such situation nicely demonstrates one of the conflicts that exist in the particular estimate of the model. Remaining two entries in the corresponding table cell state “ ρ_{a^*} higher: i^* ” and “ ρ_{a^*} lower: y^*, π^* ”. These entries demonstrate a conflict right away: i^* would prefer higher

values of parameter ρ_a than its posterior values and observables y^*, π^* would prefer lower values of the same parameter.

Another group of conflicts that deserve mentioning is group of AR1 parameters in all models with VAR foreign economy (V1-V4). It is not unusual for such AR1 processes to demonstrate this type of behavior. The series that is described in an autoregressive manner often prefers different value of the AR1 parameter then the rest of the model.

Generally, as Table 2 and Table 3 show, Monacelli-foreign models present greater variability in trade-off, a lot of different parameters bear trade-offs for fit. Furthermore, if we neglect AR1 parameters in VAR-foreign economies, Monacelli-foreign models have much higher count of trade-offs.

As for the parameters of importance (habit persistence h and price indexation parameters δ_H and δ_F), h creates trade-offs in model M2, whereas δ_F creates trade-offs in models M3, M4, V3 and V4, that is, in all models where δ_F is allowed to be non-zero. The fact that price indexation creates trade-offs for fit wherever it is allowed to be non-zero seem rather to spoil model fit than improve it. Habit persistence is in this sense much better, since it bears only one trade-off in four models, where it is allowed to be non-zero.

2.2.1 Example of results from Ratto's GSA toolbox

An example of mapping-the-fit result for M1 and five selected parameters are depicted in Picture 2.

The procedure of computation mapping-the-fit results is carried out as follows: (1) Structural parameters are sampled from posterior distribution, (2) RMSE (root mean squared error) of 1-step-ahead prediction is computed for each of observed series, (3) 10 % of lowest RMSE is defined as behavioral and B is defined as a subset of parameter values producing these behavioral results and (4) the calculations result in a number of distributions $f_j(X_i | B)$ that represent the contribution of parameter X_i to best possible fit of j -th observed series.

Plotting the distributions (or better the CDFs) is one step further to trace possible trade-offs. A trade-off is present, when at least two distributions differ from posterior distribution (denoted in Picture 2 as base) and differ from each other.

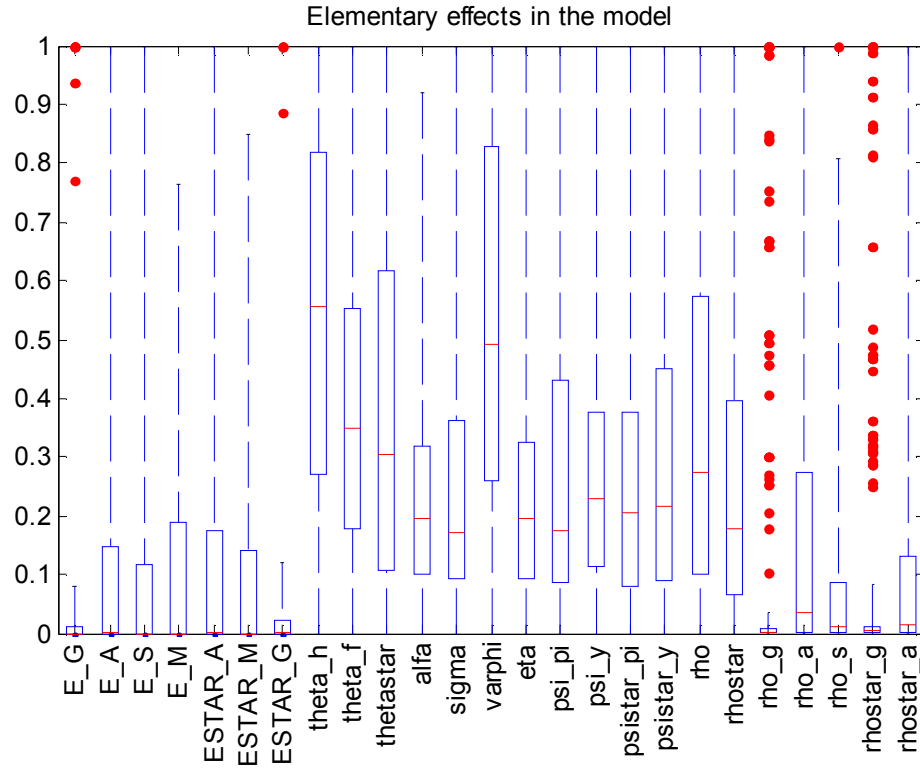
Posterior mode (base) is depicted with black dotted line. Observables causing biggest trade-offs or conflicts are in bold. These conflicts can be found in Table 2 and are interpreted above. Similar pictures were drawn for all parameters in all eight models, the results are summarized in Table 2.

2.3 Parameter importance

Results discussed in this section are outcomes of a part of GSA called Elementary Effects. For more detailed narrative on the topic, see see Saltelli (2008), Ratto (2008) or Čapek (2009). Results for our eight models are in Table 2 and Table 3, column 5.

Elementary effects analysis can identify the most and the least important parameters in a model by investigating all possible relationships in the model and identifying, which

parameter is un/important for that particular relationship. Parameters that are important play a significant role in many relationships among variables and in some they play a major role. Parameters that are unimportant may play major role for few relationships in the model and are virtually useless for explanation of most model relationships.



Picture 3: Example: Elementary effects in model M1

Not surprisingly, parameters that are unimportant are mostly AR1 parameters (ρ s and ω s), since they are usually only in one equation, which is not too interconnected with other equations of the system. Parameters that are most important in the models are θ_H , ρ_i , θ_F , h , and φ . θ s are shares of non-optimizing agents, h is habit persistence parameter, φ is inverse elasticity of labor supply and ρ_i is backward-looking parameter in monetary rule. Price-setting of agents in domestic and foreign economies is therefore important part of the model. Habit persistence is important too (when allowed).

2.3.1 Example of results from Ratto's GSA toolbox

Example of GSA result for model M1 is in Picture 3. In this case, the theory underlying the results is called Elementary Effects. These Effects are normalized measures of sensitivity of output to different input changes. In case that the input change is fully recognized by the output, the normalized effect is 1. In case that the input change doesn't change output at all, the normalized effect is 0. The domain is searched for elementary effects by Morris sampling algorithm, which creates many elementary effects for each parameter (input).

Elementary effects are summarized with boxplots with the following meaning: Lower bound of the box is lower quartile, upper bound of the box is upper quartile, central red line

denotes median, dashed lines are whiskers, which span to values not considered outliers and red dots are outliers. In Picture 3, two parameters with boxplots placed closest to the top of the picture are θ_H and φ . These parameters therefore represent most important relationships in the model. Parameters with boxplots barely visible around zero are ρ_g and ρ_{g^*} . Above these little boxplots there are a lot of red dots, parameters therefore represent a few important relationships but most relationships concerning these parameters are unimportant.

Similar pictures were drawn for all eight models, the results are summarized in Table 2.

3 Data fit and prediction quality

This section addresses the fit of the time series of the models without utilizing GSA toolbox. It conducts an analysis of fit of all observable time series and analysis of quality of prediction in these series.

3.1 Root Mean Squared Errors of one-step-ahead forecasts

Table 4 demonstrates values of RMSE (Root Mean Squared Error) of a one-step-ahead prediction, which can be considered a measure of quality of prediction and also a quality of data fit. Best result (lowest RMSE) are indicated by a star (*), worst results are indicated by a dagger (†). Note that models V1-V4 demonstrated almost the same results for foreign economy, because foreign economy is described by simple AR1 processes (marked with gray shading) that are very loosely interconnected with the rest of the model.

variable	M1	M2	M3	M4	V1	V2	V3	V4
inflation	2.86†	2.59	2.50	2.30	2.77	2.48	2.40	2.14*
output	0.62†	0.35	0.62†	0.34	0.53	0.31*	0.54	0.31*
interest rate	0.27	0.30	0.28	0.32†	0.25*	0.25*	0.27	0.27
real exch. rate	1.46	1.58†	1.33*	1.52	1.35	1.50	1.33*	1.44
foreign int. rate	0.23	0.27	0.25	0.28†	0.11*	0.12	0.11*	0.12
foreign inflation	1.20	1.21	1.06*	1.07	1.39†	1.39†	1.39†	1.39†
foreign output	0.23	0.21	0.24†	0.20	0.10	0.09*	0.10	0.10

Table 4: Root Mean Squared Errors of one-step-ahead forecasts

In the sense of comparing RMSEs, the most successful model is V2 and V4 (both demonstrate two best predictions among the models – not counting grayed area). The least successful models are M1 and M4, both demonstrating two worst prediction results among the models. Models with foreign sector modeled as three AR1 processes therefore seem to predict generally better than models with structural foreign sector.

3.2 Root Mean Squared Errors of smoothed shocks

Table 5 shows results of RMSEs calculated from smoothed shocks of the models. Comparability of smoothed shocks is limited if they enter the model in a different way. Foreign sector results in M models and V models are therefore not comparable (for V models, the cells are highlighted with gray shading). In this context, worst model is M1 with three worst results. Best model is hard to find because of limited comparability, but there are some candidates: M4

exhibits 3 best results (lowest RMSEs) and two worst results. V1 exhibits two best results and one worst (not counting data in grey area).

shock	M1	M2	M3	M4	V1	V2	V3	V4
ζ_a	12.34	14.00	13.69	16.60†	12.76*	13.37	13.61	14.02
ζ_g	2.41	1.38	2.41	1.36*	3.18†	1.91	3.12	1.85
ζ_M	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
ζ_s	0.84†	0.82	0.74	0.77	0.63*	0.67	0.63*	0.64
ζ_i^* / ζ_M^*	0.11†	0.10*	0.11†	0.10*	0.12	0.12	0.12	0.12
ζ_π^* / ζ_a^*	12.01	15.02	11.86*	15.77†	0.41	0.41	0.41	0.41
ζ_y^* / ζ_g^*	1.14†	0.50	1.10	0.49*	0.44	0.44	0.44	0.44

Table 5: Root Mean Squared Errors of smoothed shocks

Conclusion

The goal of the paper was to assess data fit of Slanicay and Vašíček (2009) model variants with different tools than posterior odds and to compare the results to findings of Slanicay and Vašíček (2009).

Section 2.1 analyzes prior space of the models and determines, which part of prior space belongs to stable model results, which part belongs to unstable results and which part belongs to indeterminacy. Subsection 2.1.2 presents a possible solution to a situation, when too small part of prior space belongs to stable results. Summarizing of results of this section shows that VAR-foreign variants have approximately 10 percentage points more stable prior space. This can (in some aspects) lead to better model results and possibly to better model fit.

Section 2.2 analyzes results of mapping-the-fit analysis. Models with structural foreign economy seem to suffer from trade-offs for fit more than models with VAR-foreign economy. This result is intuitive, since models with structural foreign economy have more mutual relationships with other equations of the model. In layman's terms, more relationships are likely to bear more trade-offs. As for the parameters of importance, price indexation seems to conflict with model fit significantly. On the other hand, habit persistence interferes with model fit only slightly.

Section 2.3 uncovers unimportant and most important parameters for eight variant models. Lists of (un)important parameters doesn't differ much among the models. Habit persistence and price indexation is a core research interest of Slanicay and Vašíček (2009). In this paper's calculations, habit persistence parameter is one of the most important parameters in models M4, V2, and V4, but parameters for price indexations (δ parameters) are not among most important parameters in any model.

Sections 3.1 and 3.2 calculate some RMSE-based indicators characterizing data fit. Summary of calculated root mean squared errors of one-step-ahead predictions and RMSEs of smoothed shocks suggests that models with VAR-foreign sector tend to err less.

Slanicay and Vašíček (2009) came to the following conclusions:

- 1) Habit persistence in consumption (in utility function) considerably increases data fit.
- 2) Inclusion of price indexation in the models decrease their data fit.
- 3) Modeling foreign sector structurally or with AR1 processes produces ambiguous results: Some model specifications favor structural foreign sector, some other favor AR1 foreign sector.

As for 1), this paper comes to similar results. Section 2.2 shows that habit persistence interferes with model fit only slightly. Section 2.3 shows that habit persistence is among the most important parameters in models where it is allowed to be non-zero.

As for 2), again this paper comes to similar results. Section 2.2 shows that price indexation always conflicts with model fit and section 2.3 shows that price indexation parameters (δ parameters) are not among most important parameters in any model.

Finally, as for 3), this paper comes to different results. Section 2.1 finds rather weak link to model fit and in that context, VAR-foreign economies perform better. Sections 3.1 and 3.2 both come to the result that in the sense of model fit, VAR-foreign models tend to err less. Contrary to the observation of Slanicay and Vašíček (2009), this paper does not find any model restriction, when Monacelli-foreign model performs better than its VAR-foreign counterpart.

Carrying out the comparison, final question pops up: Why the difference? Answering the question is not an easy task, since methodologies of this paper and Slanicay and Vašíček (2009) differ considerably. It may be a topic for further research.

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